Second Chance

Britannica Mathematics in Context

Data Analysis and Probability

HOLT, RINEHART AND WINSTON

Mathematics in Context is a comprehensive curriculum for the middle grades. It was developed in 1991 through 1997 in collaboration with the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands, with the support of the National Science Foundation Grant No. 9054928.

This unit is a new unit prepared as a part of the revision of the curriculum carried out in 2003 through 2005, with the support of the National Science Foundation Grant No. ESI 0137414.

National Science Foundation

Opinions expressed are those of the authors
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Bakker, A., Wijers, M., and Burrill, G. (2006). *Second chance*. In Wisconsin Center for Education Research & Freudenthal Institute (Eds.), Mathematics in Context. Chicago: Encyclopædia Britannica, Inc.

ISBN 0-03-038558-X

1 2 3 4 5 6   073   09 08 07 06 05

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Development 2003–2005

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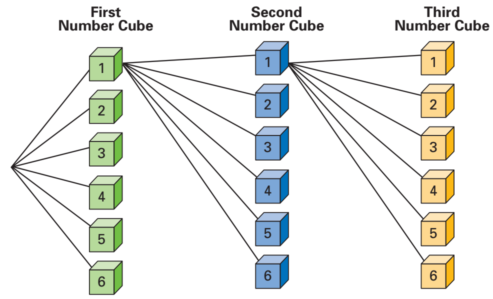
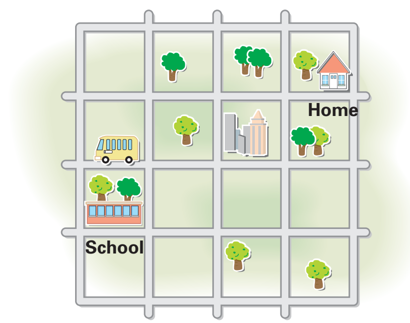


First Number Cube
Second Number Cube
Third Number Cube



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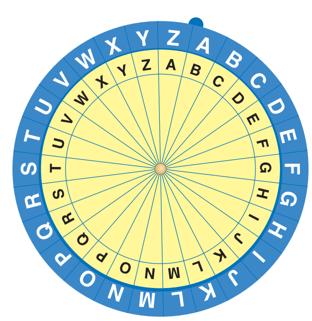


Dear Student

One thing is for sure: Our lives are full of uncertainty. We are not certain what the weather tomorrow will be or if we are going to win a game. Perhaps the game is not even fair!

In this unit you learn to count possibilities in smart ways and to do experiments about chance. You will also simulate and compute chances. What is the chance that a family with four children has four girls? How likely is it that the next child in the family will be another girl? You will learn to adjust the scoring for games to make them fair.

Sometimes information from surveys can be recorded in tables and used to make chance statements.

Chance is one way to help us measure uncertainty. Chance plays a role in decisions that we make and what we do in our lives! It is important to understand how chance works!

We hope you enjoy the unit!

Sincerely,

The Mathematics in Context Development Team

A Make a Choice

Make a Choice

Here are Robert's clothes.

1. How many different outfits can Robert wear to school? Find a smart way to count the different outfits.

Hillary says to Robert, "If you pick an outfit without looking, I think the **chance** that you will choose my favorite outfit—the striped shirt, blue pants, and tennis shoes—is one out of eight!"

2. Is Hillary right? Explain why or why not.

3. **a.** Which of the statements Robert makes about choosing his clothes are true?

   i "If I choose an outfit without looking, the chance that I pick a combination with my striped shirt in it is four out of 16."

   ii "If I choose an outfit without looking, the chance that I pick a combination with my tennis shoes in it is two out of 16."

   iii "If I choose an outfit without looking, the chance that I pick a combination with both my tennis shoes and my striped shirt is one out of eight."

   b. Write a statement like the ones above that Robert might make about choosing his clothes. Your statement should be true and begin with, "If I choose an outfit without looking, the chance that I pick…."

4. a. How many different outfits can Robert wear if he buys another pair of pants?

   b. If he buys another pair of pants, how does the chance that Robert picks Hillary's favorite outfit (striped shirt, blue pants, and tennis shoes) change? Explain.

A Class Trip

Grade 7 in Robert's school is planning a two-day class trip to a lake for a science field trip.

They can choose to go to one of four lakes: Lake Norma, Lake Ancona, Lake Popo, or Lake Windus.

Besides choosing the lake, the class has to choose whether to camp out in a tent or to stay in a lodge and whether to take a bus tour around the lake or a boat trip.

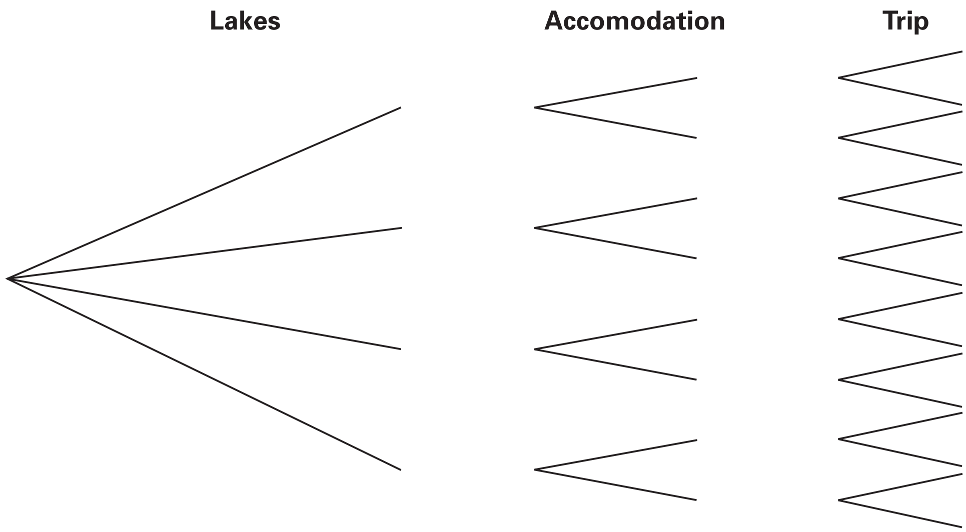
The class has to make a lot of decisions!

5. a. Finish the **tree diagram** on **Student Activity Sheet 1**. Write the right words next to all the branches in the tree.

   b. **Reflect** How many different class trips are possible for Robert's class to choose?

   c. How does this problem relate to the problem about the different outfits Robert can choose?

   d. How many possibilities are there if Robert's class does not want to go camping?



Robert's class finds it hard to decide which trip to choose. Different students like different options. Fiona suggests they should just write each possible trip on a piece of paper, put the pieces in a bag, and pick one of the possible trips from the bag.

6. a. If Robert's class picks one of the trips from the bag, what is the chance that they will go camping?

b. What is the chance they will go to Lake Norma?

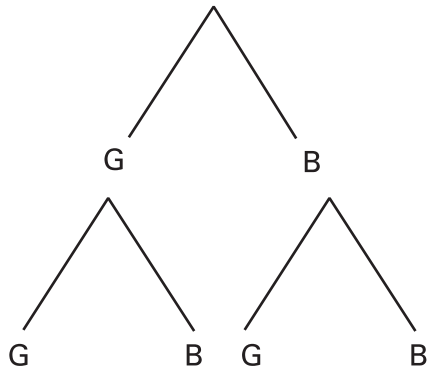
Families

Nearly as many baby girls as baby boys are born. The difference is so small you can say that the chance of having a boy is equal to the chance of having a girl.

Sonya, Matthew, and Sarah are the children of the Jansen family. A new family is moving into the house next to the Jansen house.

They already know that this family has three children about the same ages as Sonya, Matthew, and Sarah. "I hope they have two girls and one boy just like we have," Sonya says, "but I guess there is not much chance that will happen."

7. Do you think the chance that a family with three children where two are girls and one is a boy will move in next door is more or less than 50%? Explain your reasoning.



The tree diagram shows different possibilities for a family with two children.

8. **a.** How many different possibilities are there for a family with two children?

b. Explain the difference between the paths BG and GB.

c. What is the chance that a family with two children will have two girls?

d. What is the chance that the family will not have two girls? How did you find this chance?

You can express a chance as a ratio, "so many out of so many," but you can also use a fraction or a percent. The chance of having two boys in a family with two children is:

one out of four.

This can be written as $\frac{1}{4}$.

This is the same as 25%.

9. **Reflect** Explain how you can see from the tree diagram that the chance of having two boys is 1 out of 4.

10. Write each of the chances you found in problems 6a, 6b, 8c, and 8d as a ratio, a fraction, and a percent.

11. **a.** In your notebook, copy the tree diagram from problem 8 and extend it to a family with three children.

b. In your tree diagram, trace all of the paths for families with two girls and one boy.

c. What is the chance that a family with three children will have two girls and one boy? Write the chance as a ratio, as a fraction, and as a percent.

12. **a.** What is the chance that a family with three children will have three boys?

b. Write another chance statement about a family with three children.

About 500 families with three children live in East Lynn.

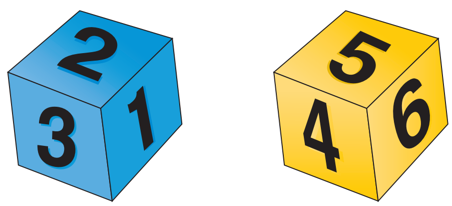
13. **Reflect** Would you be surprised if 70 of these 500 families with three children had three boys? Explain.

You can often find a chance by calculating:

$$\frac{\textbf{the number of favorable outcomes}}{\textbf{total number of possible outcomes}}$$

Activity

Number Cubes



Roll two number cubes of different colors 10 times.

List the combination you rolled, like "blue 2 and yellow 5."

Work with three other classmates and list all of your outcomes. Find a systematic way to make your list.

14. Did the four of you roll all possible combinations of the two number cubes? Explain how you decided.

You can use tree diagrams to count all possible outcomes of an event. Sometimes you can count all the outcomes by using a chart. For example, if you want to see all possibilities when throwing two number cubes—a blue one and a yellow one—you can use this table.

	1	2	3	4	5	6
1	1 1	1 2	1 3	1 4	1 5	1 6
2	2 1	2 2	2 3	2 4	2 5	2 6
3	3 1	3 2	3 3	3 4	3 5	3 6
4	4 1	4 2	4 3	4 4	4 5	4 6
5	5 1	5 2	5 3	5 4	5 5	5 6
6	6 1	6 2	6 3	6 4	6 5	6 6

Max rolled two number cubes. On **Student Activity Sheet 2**, you see a circle marking the combination Max rolled.

15. a. What combination did Max roll with the number cubes? What is the sum of the two number cubes he rolled?

b. Brenda rolled the same sum as Max, but she did not roll the same combination. In the first chart on **Student Activity Sheet 2,** circle all combinations Brenda may have rolled.

c. In the table at the bottom of **Student Activity Sheet 2**, write the sum for each combination of rolling two number cubes.

Brenda thinks the chance of rolling a sum of eight with two number cubes is the same as the chance of rolling a sum of three. She reasons:

> With two number cubes you can roll a sum of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12. This makes 11 possibilities in total, so the chance for each of these outcomes is one out of eleven, which is the same as $\frac{1}{11}$, or about 9%.

16. a. Do you agree with Brenda? Why or why not?

b. **Reflect** What is the chance that you will roll a sum greater than 8 with two number cubes?

Jackie says that the chance of rolling a sum of either 9, 10, or 11 with two number cubes is 25%. Tom says, "No, the chance is 9 out of 36."

17. a. Is Jackie right? Explain.

b. What would you say to Tom?

18. a. Why is a chart like the one shown before problem 15 not useful for listing all possibilities when throwing three number cubes?

b. What is the total number of possible results when throwing three number cubes? How did you find this?

Codes

You need a code to open some school lockers as well as to access ATM machines and often to open garage doors. A four-digit code is used for the garage door at Brenda's home. The code is made up of numbers from zero to nine. All of the numbers can be used more than once.

For security reasons, if a wrong code is used three times in a row, the garage door will stay locked for the next half hour.

Brenda's brother is at the garage door, but he forgot the code. He only remembers it starts with 3–5, and he knows for sure there is no 0 in the code.

So the code is:

3 5 — — (no zeros)

He decides to guess.

19. a. What is the chance that his first guess is correct?

b. Suppose the first guess is wrong. He keeps on guessing. How likely do you think it is that he guesses wrong and the door will remain locked for a half hour?

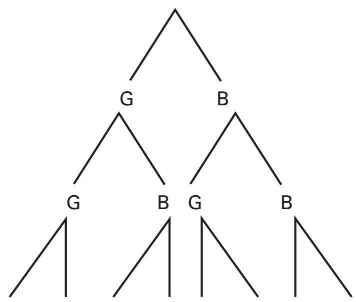
Suppose the code for the garage door consists of four letters instead of four numbers, and Brenda's brother remembers only the first two letters of the code.

20. How will this change the chance that the garage door will remain locked for half an hour?

Make a Choice

Summary

If you want to count the possible ways that something can occur you can:



- draw all different combinations as you did for Robert's clothes;
- find a smart way to write down all possibilities, such as GG GB BG BB (G for girl, B for boy) for a family with two children;
- use a tree diagram like the one showing the possibilities for the families with three children;
- use a table such as the one for tossing two number cubes;
- use smart calculations as you did for the codes.

If you know all possible outcomes, and you know all outcomes have the same chance of occurring, you can make statements about the chance that certain outcomes may occur. You can do this by counting how many times this outcome occurs compared to all possible outcomes. The chance is:

$$\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

For a family with two children, the four different outcomes GG, GB, BG, and BB are equally likely. Two of those outcomes have a boy and a girl. Therefore, the chance of having a boy and a girl in a family of two children is two out of four, or one out of two.

You can express a chance either as a ratio, like "two out of four;" as a fraction, $\frac{2}{4}$, which is the same as $\frac{1}{2}$; or as a percent, 50%.

Think back to the trip Robert's class is planning.

Check Your Work

1. a. How can you calculate—without drawing a tree diagram—how many possible trips Robert's class can choose? (See page 2.)

   Robert says, "The chance we will go on a boat trip is $\frac{8}{16}$."
   Noella says, "I think this chance is 1 out of 2, or 50%."

   b. Comment on Robert's and Noella's statements.

Mario's advertises, "We serve over 30 different three-course meals."

Customers can choose soup or salad as an appetizer; fish, chicken, beef, or a vegetarian dish for the main course; and fruit, ice cream, or pudding for dessert.

2. Do you think Mario's advertisement is correct? If yes, show why. If no, give an example of a number of appetizers, main courses, and desserts that will lead to more than 30 different meals.

Diana is having her birthday dinner at Mario's. She decides to make a surprise meal for herself by choosing each of the courses by chance.

3. a. What is the chance that Diana has a meal with soup and beef?

   b. What is the chance that Diana has a meal without fish?

Diana does not like pudding. She thinks the chance that she will pick a meal with pudding for dessert is very small. She says, "The chance that I will pick pudding in my surprise meal is only one out of 24."

4. a. Do you agree with Diana? Explain your answer.

   b. How many meals are possible if pudding cannot be chosen?

For Further Reflection

Explain how finding the chance of an outcome using a tree diagram is related to finding the chance using the rule:

$$\text{chance} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

You may use an example in your explanation.

B

A Matter of Information

Car Colors

Sometimes chances can be found because you know and can count all possible results or outcomes. You saw examples of this in Section A.

In other situations, you can make statements about chance by collecting information about the possible outcomes.

Cars come in different colors. Some car colors are more common than others.

1. **a.** If you go out on the street where you live, what color car do you expect to see most?

b. Do you think all of your classmates will have the same answer for **a**? Why?

Janet and her sister Karji discuss car colors. Janet says that the favorite color for cars in their neighborhood is white because white cars are easy to see on roads. Karji argues that red is more common because red is a lot of people's favorite color. To find out who is right, Janet and Karji record the colors of 100 cars in a parking lot nearby.

The results are in the table.

Color	Number of Cars
Red	13
White	24
Other	63
Total	**100**

2. **a.** By looking at the results in the table can you tell who is right—Janet or Karji? Explain.

   b. Which chance do you think is bigger—that the first car leaving the parking lot is red or that the first car leaving the lot is white? Why?

Activity

As a class, you are going to investigate car colors in a parking lot or on the street.

First agree on four colors you want to record. Record cars that are not one of those four colors as "other."

Design a form on which you can record the car colors.

Record the colors of 25 different cars. Try to choose a different set of cars from ones chosen by others in your class.

3. **a.** Combine the class results in one table. Make a graph of the results.

   b. Calculate the percentage of cars in each color.

Suppose all of the cars the class tallied in the activity were from the same parking lot.

4. **a.** Which color car are you most likely to see leaving the parking lot?

   b. Is it possible that the first car entering the parking lot the day after you counted colors is a color that you did not record in the activity? Explain your answer.

   c. Write three statements involving chance based on your findings about car colors.

A Word Game

Brittney and Kenji are playing a word game. Brittney is guessing a word that Kenji is thinking about. Kenji makes a row of ten dots, one dot for each letter in the word he has in mind.

Now Brittney has to guess a letter. If the letter is correct, Kenji puts the letter over the correct dot (or dots) in the word. If the letter is not in his word, Kenji writes it down.

Brittney wins if she guesses the correct word before she has guessed eight "wrong" letters. Kenji wins if Brittney guesses eight letters that are **not** in the word and still hasn't guessed his word.

Brittney first asks if the letter E is in the word.

5. Why do you think Brittney first chooses the letter E?

Kenji writes down: **. E. . E . . . . E**

Brittney tries A, O, I, and U.

Kenji wrote: **. E . . E . . A . E** **wrong: O I U**

6. a. What do you think would be a good letter to ask about next? Why do you think so?

b. Guess the word or finish the game. (Your teacher has the answer!)

Letter Frequency

Not all languages use the same letters equally often.

7. a. Which letter do you think is used most frequently in the English language?

   b. Which letter do you think might occur the least often in the English language?

This table shows the average letter **frequency** in typical written English.

English Letter Frequency			
Letter	**%**	**Letter**	**%**
a	8.17	n	6.75
b	1.49	o	7.51
c	2.78	p	1.93
d	4.25	q	0.10
e	12.70	r	5.99
f	2.23	s	6.33
g	2.02	t	9.06
h	6.09	u	2.76
i	6.97	v	0.98
j	0.15	w	2.36
k	0.77	x	0.15
l	4.03	y	1.97
m	2.41	z	0.07

   c. How close were your answers for parts **a** and **b**? What are the most and least used letters according to this table?

8. **Reflect** If you know how frequently letters are used in writing, do you think this will help you when playing the Guess My word game? Why or why not?

Activity

Take a newspaper article or a text from any book. With a classmate, record the first 100 letters in this text in a frequency table.

9. Use **Student Activity Sheet 3a** to make a graph of the frequencies for each letter.

10. **a.** What is the most common letter in your selection? Was this the same for every pair of students in your class?

    b. Compare your graph with your classmates' graphs. What do you notice?

11. **a.** On **Student Activity Sheet 3b**, combine the letter frequency graphs you made in problem 9 into one class graph.

    b. Write three lines comparing the graph to the letter frequency table.

The results of an experiment or data collection can be used to estimate the chance an event will occur. Chances found this way are called **experimental chances**.

12. Use the data in your frequency table from problem 11 to answer the following:

    a. If you close your eyes and select a letter from a newspaper, estimate the chance that you pick an O.

    b. As a class, compare your answers in part **a** by making a dot plot on the number line of the estimated chances. Use the plot to help you write a sentence about the probability of selecting the letter O from a newspaper with your eyes closed.

    c. Estimate the chance of picking three other letters. Choose one with a high probability of being picked and another with a low probability of being picked.The third one can be any letter you want. Write each answer as a fraction and as a decimal.

Family Dining

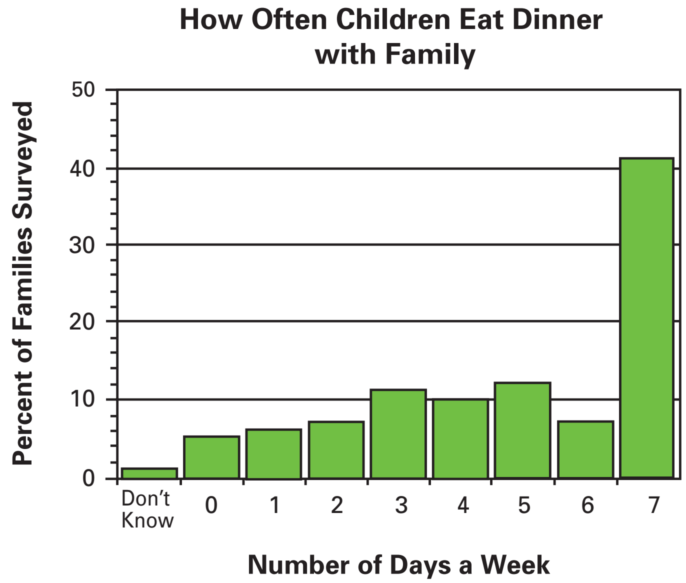
Instead of collecting information about possible outcomes yourself in order to make chance statements, you often can use information collected by others.

Do students often have dinner with their families? Researchers were interested in answering this question. They surveyed students aged 12 to 17, and the results are in the table below.

Number of Days a Week Children (Age 12–17) Have Dinner with Their Family	Percentage
Don't know	1%
0	5%
1	6%
2	7%
3	11%
4	10%
5	12%
6	7%
7	41%

Source: National Center on Addiction and Substance Abuse

These results can be graphed:



13. a. If the researchers interviewed 3,000 families, how many reported eating together more than five days a week?

b. If one of the families in the study is picked at random, what is the chance that the family eats together more than five days a week?

c. Use your answer for part **b** to find out what the chance is that a family in the study picked at random eats together five days a week or fewer.

14. a. Is the chance that a family eats together seven days a week greater than, the same as, or less than the chance that they do fewer than five days a week? Explain how you found your answer.

b. What is the chance that a family does not eat together two days a week?

Who Wears Glasses?

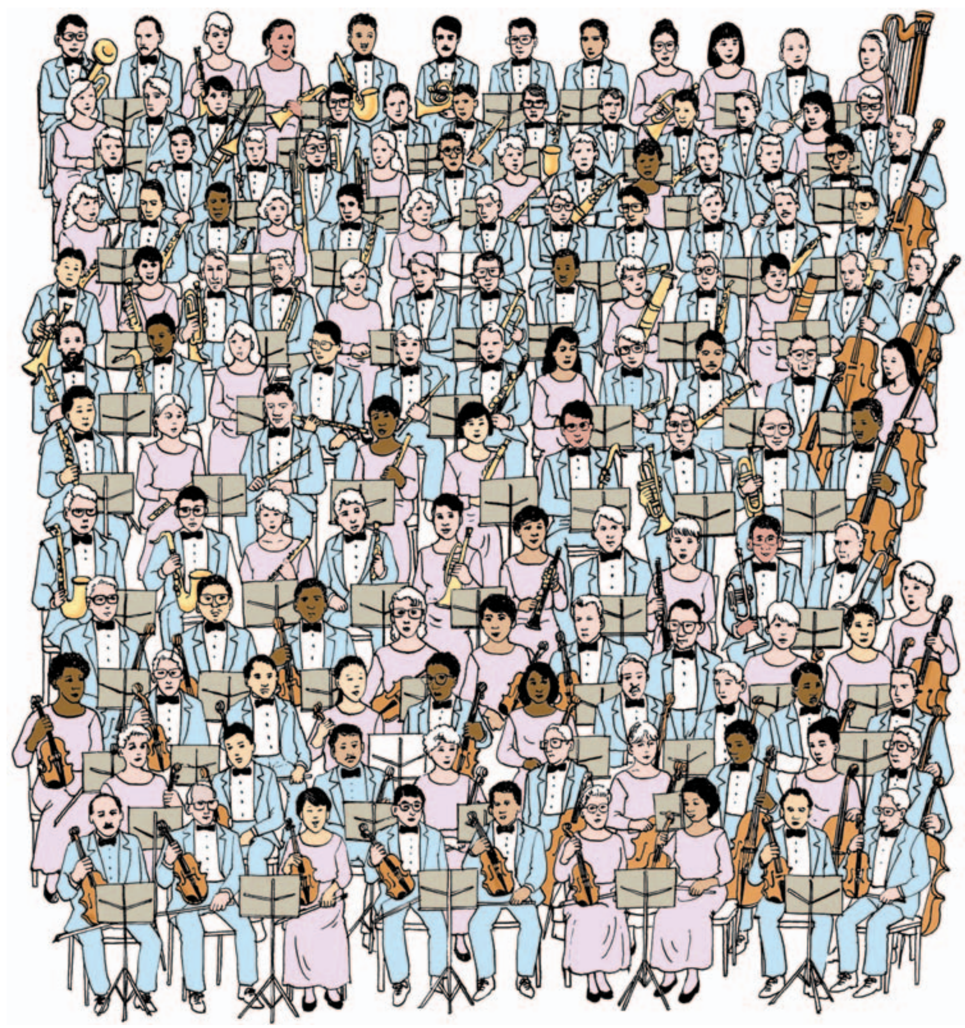
Some people wear glasses, and some people don't. It is not easy to estimate what the chance is that the first person you meet on the street will be wearing glasses.

Joshua announced that he thinks more men than women wear glasses.

15. **a.** What do you think about Joshua's statement?

b. How could you figure out whether or not it is true that men are more likely to wear glasses than women?

This illustration was used in an advertisement for an orchestra.



16. Use the illustration to decide whether men or women in the orchestra seem to be more likely to wear glasses. Explain how you came to your conclusion.

Counting men and women with and without glasses can tell you—for those you counted—how many men and women wear glasses. But counting just the number who wear glasses cannot tell you what the chance is that a person wears glasses.

17. Suppose that you randomly select a man from the orchestra. Estimate the chance that this man wears glasses. Explain how you made your estimate.

The **two-way table** summarizes the information about whether or not the musicians in the illustration wear glasses.

	Men	Women	Total
Glasses	32	3	35
No Glasses	56	39	95
Total	88	42	130

A member of the orchestra is chosen at random.

18. **a.** What is the chance that the person chosen wears glasses?

b. If you were told that the person is a woman, would you change your answer for part **a**?

Chance can be expressed in different ways.

You can express a chance as a ratio, like 35 out of 130.

You can use a fraction, like $\frac{35}{130}$.

You can use decimals or percents such as $\frac{35}{130} \approx 0.269 \approx 27\%$.

Sandra states, "The chance that a randomly chosen woman in the orchestra does not wear glasses is 39 out of 42, which is almost 100%."

Juan states, "I don't agree. The chance that a randomly chosen woman in the orchestra does not wear glasses is 39 out of 130, which is only 30%."

19. **a.** Explain how Sandra and Juan may have reasoned.

b. Do you agree with Sandra or with Juan? Explain your thinking.

Look at the musicians in the illustration again.

20. How many musicians could you draw glasses on to make it appear that "wearing glasses is as likely for men as for women"?

Watching TV

The seventh grade class in Robert and Hillary's school surveyed all of the students in grades 7 and 8 to find out how much television they watched each day. Some of their results are in the two-way table.

21. **a.** Finish the table of Robert and Hillary's survey.

	Less Than 3 Hours of TV per Day	3 Hours or More of TV per Day	Total
Grade 7		35	
Grade 8			40
Total	**50**	**50**	

b. Is there a difference between the number of hours students in grade 7 and students in grade 8 watch TV?

22. **a.** What is the chance that a student chosen at random from Robert's school watches three hours or more of TV a night?

b. If you knew that the student was in grade 7, would you change your answer for part **a**? Explain why or why not.

c. If you find a student who watches TV more than three hours a night, what is the chance that this student is in grade 8?

Math History

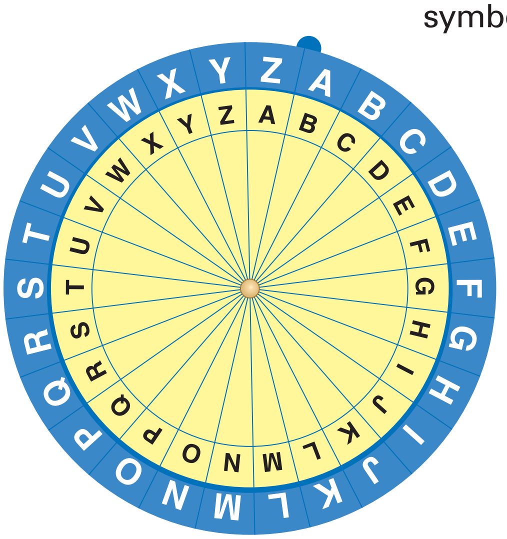
Frequency Analysis

Knowing letter frequencies is useful for winning games like Hangman and also for cracking codes.

In the ninth century, an Arabian scientist named Al-kindi wrote about a method for code-breaking now known as frequency analysis. He discovered that the variation in frequency of letters in a document can be used to decipher encrypted text. This is a translation of some Al-kindi text taken from *The Code Book* by Simon Singh.

> *One way to solve an encrypted message, if you know its language, is to find ordinary text of the same language long enough to fill one sheet or so, and then count the occurrences of each letter. You can call the most frequently occurring letter the "first." The next most occurring letter the "second," the following most occurring letter the "third," and so on, until you have used all the different letters in the sample.*

Then we look at the coded text we want to solve, and we also classify its symbols. We find the most occurring symbol and change it to the form of the "first" letter of the plain text sample; the next most common symbol is changed to the form of the "second" letter; and the following most common symbol is changed to the form of the "third" letter; and so on, until we account for all symbols of the cryptogram we want to solve.



You can use this frequency analysis method of Al-Kindi to decipher the following encrypted English text!

KL, KHUH L KDYH D VKRUW WHAW IRU BRX WR GHFLSKHU, L JXHVV BRX FDQ GHFUBSW LW.

You may make your own encrypted texts using a device like this.

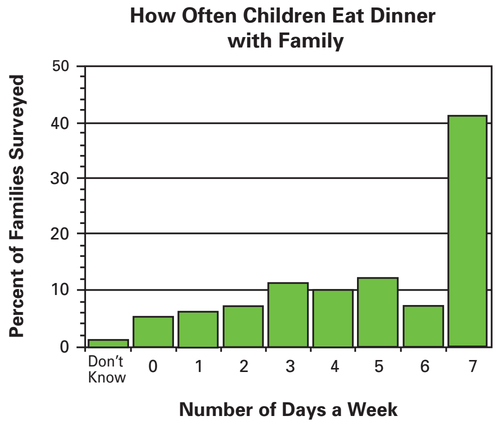
B A Matter of Information

Summary

You can collect information on how often certain outcomes occur. This information can be presented in tables or graphs.

The information can be collected for all of the cases being studied, as in asking everyone in Robert's school how many hours they watched television.

You can use this information to state the chances of certain outcomes for those cases.



Sometimes, not all of the information is available and you have to take a sample as in counting the number of times the letters of the alphabet are used in a newspaper. When this is the case, you can only estimate the chances of an outcome.

When chance is estimated from experiments or surveys, it is sometimes called experimental chance.

If the chance is known before collecting data, like for tossing number cubes or coins, you can call this the theoretical chance.

Figuring the chance that an event occurs depends on what you know.

For example, if you know a person from Robert's school is in 7th grade, you only use the information about the 7th grade to make chance statements and not any of the information about the 8th grade.

You can record results of related outcomes in a two-way table and use the information in the table to make chance statements.

	Men	Women	Total
Glasses	32	3	35
No Glasses	56	39	95
Total	**88**	**42**	130

Check Your Work

Ages of Doctors in the United States

Use the information in the table to answer the following questions.

Ages of Doctors in the United States			
Doctors	**Female**	**Male**	**Total**
Under 35 Years Old	60,000	80,000	140,000
35 Years and Over	150,000	550,000	700,000
Total	210,000	630,000	840,000

Source: American Medical Association, December 31, 2001

1. a. If you choose a doctor at random, estimate the chance that the doctor will be female.

b. Was there a difference in the chance that a randomly chosen doctor would be female rather than male ten or twenty years ago? Explain your thinking.

c. If you randomly choose from a set of doctors you know to be under 35, what is the chance that the doctor will be a male?

d. If you choose a doctor at random from those you know to be male, what is the chance that the doctor will be older than 35?

e. What observations can you make about the chance that a doctor will be young or old and be male or female?

Robert's mother has to replace three keys on her computer keyboard because the letters on them had worn off.

2. **Reflect** Which three letters do you think she had to replace? Explain your answer.

B A Matter of Information

Middle school and high school students in the Parker School District were asked whether or not they had seen a recent movie.

	Saw Movie	Did Not See Movie	Total
Middle School	34	...	100
High School	...	...	...
Total	120	...	200

3. a. Copy the table and fill in the missing information.

b. Describe the difference between middle school and high school students with respect to seeing the movie.

c. If you talked to a student who was surveyed about the movie and he told you that he hadn't seen it, what is the chance that this student is in middle school?

Blood Type and Rhesus Factor (RH)	Percentage of Population
O positive	36%
O negative	6%
A positive	38%
A negative	6%
B positive	8%
B negative	2%
AB positive	3.5%
AB negative	0.5%

People have four different blood types: A, B, AB, and O.

For each type, the Rhesus factor (RH), a substance in red blood cells, may be positive or negative. In the table, you see the percentage of the U.S. population with each type of blood.

4. a. If a person is selected randomly, what is the chance that this person's blood is type B?

b. What is the chance that a randomly selected person is RH positive?

c. How would the answer to **b** change if you knew the person has type B blood?

For Further Reflection

The two-way tables in this section used two sets of information like age and hours watching TV or gender and wearing glasses. Could you make a table if you had three sets of information, like age, gender, and number of hours watching TV? Explain how you could do this or why it is not possible.

C In the Long Run

Heads in the Long Run

You can reason about the chance of some events, like tossing a die, by knowing about the possible outcomes. Sometimes you can collect information from a survey and estimate the chances. Another way to think about chance is to try the situation over and over and use the results to estimate the chance that certain outcomes will occur.

Suppose you toss a coin lots and lots of times. What will happen to the chances of getting heads? The table shows the results of tossing a coin in sets of 25s.

Total Number of Tosses	Number of Heads in This Set of 25 Tosses	Total Number of Heads So Far	Chance of Getting Heads
25	16	16	$\frac{16}{25} = .64$
50	12	28	$\frac{28}{50} = .56$
75	11	39	
100	8	47	
125	13	60	
150	14	74	
175	13	87	
200	12	99	
225	12	111	
....	....	....	

1. **a.** Toss a coin 25 times and add your count to the table. Copy the table into your notebook.

   b. Toss the coin another 25 times and add the count to the table.

   The estimated chance of getting heads is the total number of heads over the total number of tosses.

   c. In the table, fill in the column **Chance of Getting Heads.**

2. a. Graph the number of tosses and the chance you will get a head on **Student Activity Sheet 4**.

   b. Describe what you see in the graph.

   c. **Reflect** Theoretically, the chances of getting a head or tail are equal. Why does the percentage of heads vary?

   d. Describe what you think will happen to the graph and the chance of getting a head if the coin is tossed 300 times more.

Deborah tosses a coin. After tossing it nine times in a row, she got this result.

H T T H H T T T T

For the tenth toss, Deborah thinks she has a much bigger chance of getting a head than a tail. Ilana says, "This is not true since the coin does not remember that it already came up tails lots of times."

3. a. Do you agree with Deborah that the chance that she will get a head on the tenth toss is bigger than the chance that she will get a tail? Why or why not?

   b. What does Ilana mean when she says that the coin does not remember?

Fair Games?

You may have played games like Monopoly® or other games that use number cubes to tell you how to proceed. Sometimes you are lucky when you play, but your success really depends on chance. A good game of chance needs to be fair; all players should have an equal chance of winning.

4. Are the following games fair? Give reasons to support your answers. You can play the games to find out!

a. Two people each flip a coin. If both coins land on the same side, A wins one point; otherwise B wins one point.

b. Two people each roll a number cube at the same time. If neither of the players roll a 5 or a 6, A wins one point; otherwise B gets one point.

c. Two players each throw two number cubes 24 times. If no double 6 occurs, A wins one point; if a double 6 occurs, B wins one point.

d. Two players take turns tossing a thumbtack. If the thumbtack lands on its back (point up), A wins; if it lands on its side, B wins.

It is not always easy to decide whether a game is fair. Sometimes you can just reason about the situation and decide whether it is fair. Sometimes you can calculate the chances, but more often you will need to play the game many times to estimate the chance of winning.

5. a. For which of the games from problem 4 could you decide whether the game was fair by reasoning or calculating? Which ones did you need to play?

b. **Reflect** Find a way to adjust the scoring so that the "unfair" games in problem 1 are fair. Explain why your scoring system will make the game fair.

The Toothpick Game

With a partner, you are going to investigate whether the Toothpick Game is fair. First you need a board for the game.

Activity

For this activity, you need:

- a toothpick,
- a ruler,
- a large sheet of paper.

Measure the length of the toothpick. Use a ruler to draw parallel lines on the sheet of paper. The distance between each two lines must be the same as the length of the toothpick. This will be your game board. (See example in **Student Activity Sheet 5.**)

Play the game with a partner. One student drops the toothpick on the game board. The other records whether the toothpick lands on a line or between two lines.

Rules:

- Decide who will be player A and who will be B.
- Put the game board on the floor. From about 18 inches above the board, drop the toothpick on the game board.
- If the toothpick lands on a line, player A gets a point.
- If the toothpick lands between two lines, player B gets a point. If the toothpick does not land on the board, take another turn.
- Take turns. Do this 50 times and record the results. The player with the most points is the winner.

6. a. Do you think the Toothpick Game is a fair game? Give reasons to support your answer.

b. If it is not a fair game, change the number of points for landing on a line and for landing between lines so the game becomes fair.

Guessing on a Test

Charlie has a geography test. The test consists of ten statements that are either true or false.

For example, one of the questions was:

Sydney is the capital of Australia.

☑ **True** ☐ **False**

Charlie has not prepared for the test, so he guesses the answers to all of the questions. He will pass the test if he gets at least seven answers correct. Charlie thinks that the chance that he will pass the test by just guessing at all the answers is more than 50%.

7. Do you agree with Charlie? If you do, explain why. If you don't, how big do you think the chance is that Charlie will pass the test when he guesses all 10 answers?

Josh says to Charlie, "We could model guessing the answers to the geography test by flipping a coin and use that to find the chance that you will pass if you guess."

8. a. What does Josh mean?

b. Could Josh and Charlie use a number cube to model "guessing the answers"? Explain.

Activity

You can toss a coin to model or simulate getting a question correct by guessing.

Heads means your answer is correct.

Tails means your answer is wrong.

Toss a coin 10 times, once for each question on the geography test, and record the results. This is like taking one test.

Flip the coin another 10 times and record the results. Do this until you have modeled taking five tests.

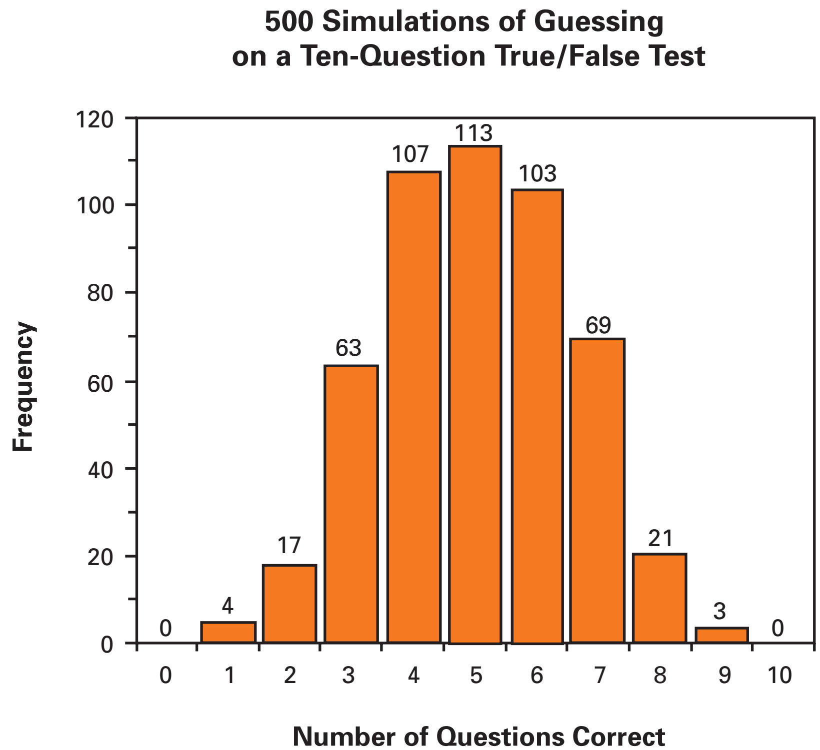
9. a. In the activity, how many answers did you "guess" correctly (that is, how many times did you throw a "head") on each of your five tests?

b. Record in a table the class results for everyone's five tests.

Number of Questions Correct	Number of Tests with This Result
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

c. Use the results in the table to estimate the chance that someone passes the test with 7 or more questions out of 10 correct by just guessing.

A graphing calculator was used to simulate guessing the answers. The calculator simulated taking the test 500 times.



10. **a.** Use the graph to estimate the chance of passing the test (seven or more correct out of 10).

b. **Reflect** Explain why the chance of having two questions on the test correct is about the same as the chance of having eight questions correct.

11. Compare the results of the **simulation** on the calculator to the results of the simulation in class. What do you notice?

Suppose the teacher increases the number of true-false questions on the geography test from 10 to 20. You pass the test if you answer at least 14 of the 20 questions correctly.

12. **a.** Do you think the chance of passing the test with 20 questions by guessing is bigger than, smaller than, or the same as passing the test with 10 problems by guessing? Describe your thinking.

b. What do you think happens to the chance of getting all the questions on the test correct by guessing if the number of questions increases from 10 to 20?

The Game of Hog

Activity

Playing the Game of Hog

Play the game of Hog 20 times with one of your classmates. You will need eight number cubes, or you can roll one cube eight times.

Here are the rules:

- Say how many number cubes you want to roll; you can choose from one to eight.
- Roll that many number cubes.
- If none of the numbers you rolled is a 1, your score is the sum of the numbers you rolled.
- If a 1 comes up on any of your number cubes, then your score is 0!
- Now it is your partner's turn.
- You can change the number of cubes you want to roll for each turn.

The object of the game is to get as large a total score as you can. For each roll, record:

- how many number cubes were used;
- how many points were scored.

Adapted from *Measuring Up*, Mathematical Sciences Education Board, National Research Council, 1993.

13. a. **Reflect** How many number cubes would it be unwise to roll when playing Hog? Explain why this is so.

b. Based on your results playing Hog, what strategy seems to give the biggest chance of winning?

14. a. How often did you have 0 points twice in a row? Did this outcome seem to depend on how many number cubes you rolled?

b. In general, how likely do you think it is to score 0 points at least twice in a row?

C In the Long Run

Summary

If you cannot compute chances of winning beforehand, you can play a game (like the Toothpick Game or Hog) or simulate the situation (like guessing on a test) many times to see what happens.

If you do many trials, the chance of an event occuring will be close to the theoretical chance. With the results from this large number of trials, you can make statements about chances.

One thing to remember about throwing number cubes and tossing coins is that number cubes and coins have no memory.

The next throw or toss is not influenced by what has happened before.

Check Your Work

1. Decide whether the following games are fair. You can play the games to find out.

   a. Two players take turns rolling a number cube; each player's score is the number rolled.

   b. Two players roll two number cubes each at the same time. If neither of the players rolls doubles, A gets one point; otherwise B gets one.

2. a. If you roll a number cube a thousand times, about how many times do you expect a 6 to come up?

b. If you roll a number cube 100 times, how often do you expect a number divisible by three? What percentage is that?

3. Melissa and Jody are playing a game. Melissa needs to roll a 6; otherwise she cannot go on. She already rolled the number cube 10 times without rolling a 6.

Jody thinks that on the next roll Melissa is almost sure to roll a 6.

a. Do you agree with Jody? Explain your reasoning.

b. Melissa thinks the number cube is not "fair." Do you agree with Melissa? Explain.

4. Peter takes a geography quiz with five multiple-choice questions. Each question has three options. Here is one of the questions.

Select the best answer.

The capital of Spain is

- ○ Barcelona.
- ○ Madrid.
- ○ Seville.

a. If Peter guesses the answer, what is his chance of guessing it wrong?

b. How can you model guessing the answer to all five questions on the test by using a number cube?

Guessing answers on this quiz was simulated 50 times. These are the results:

Number of Questions out of Five Correct	Number of Times This Occurred
0	9
1	16
2	13
3	5
4	7
5	0

c. Based on these results, what is the chance that Peter will get three or more questions on the quiz correct by guessing?

For Further Reflection

Think of a situation different from the ones in this section where you would have to simulate the situation many times to estimate the chance of an outcome. Think of another situation where you could figure out the theoretical chance and would not need to simulate the situation.

D Computing Chances

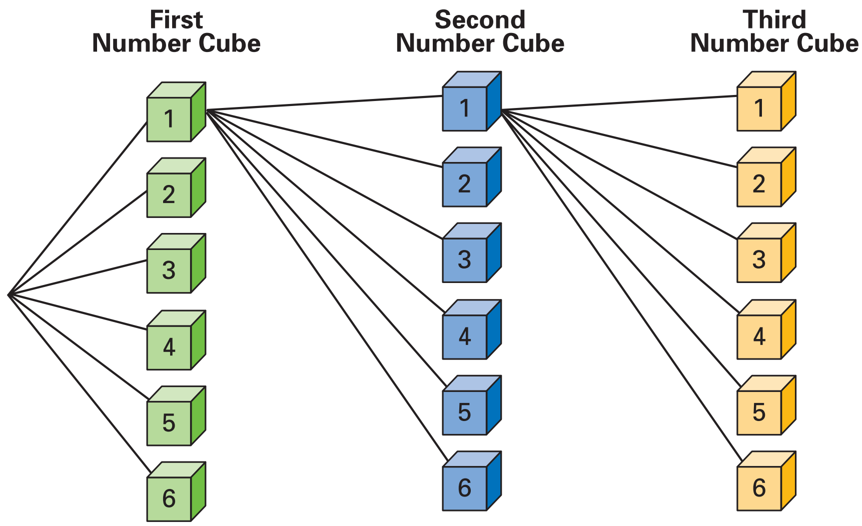
The Game of Hog Again

Sometimes you are concerned about the chance of more than one event happening. You might want to know the chances of first one thing and then another or maybe the chance of several things happening at the same time. How can you find the chance of a **combined event**?

For example, with the game of Hog you know that the chance of rolling a 1 with one number cube is one out of six, or $\frac{1}{6}$. But what about getting none, one, or two 1s when tossing two number cubes? Or more number cubes?

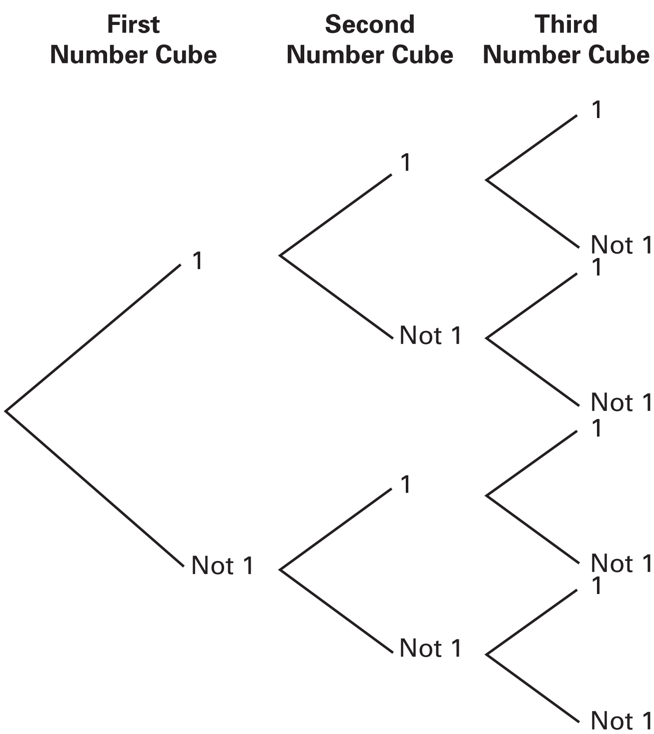
Akio is playing Hog and has rolled three number cubes.

Aiko wants to find the chance of not throwing a 1 with three number cubes. He decides to use a tree diagram to do this. This is part of his tree diagram.



1. a. Describe what Akio's complete tree diagram for rolling three number cubes will look like. Note: You do not have to draw the complete diagram.

   b. Describe how you could use the complete tree diagram to find the chance of rolling no 1s with three number cubes.



Celia thinks Akio can use a simpler tree diagram since the only important outcomes are "1" or "not 1." She suggests using a diagram like this.

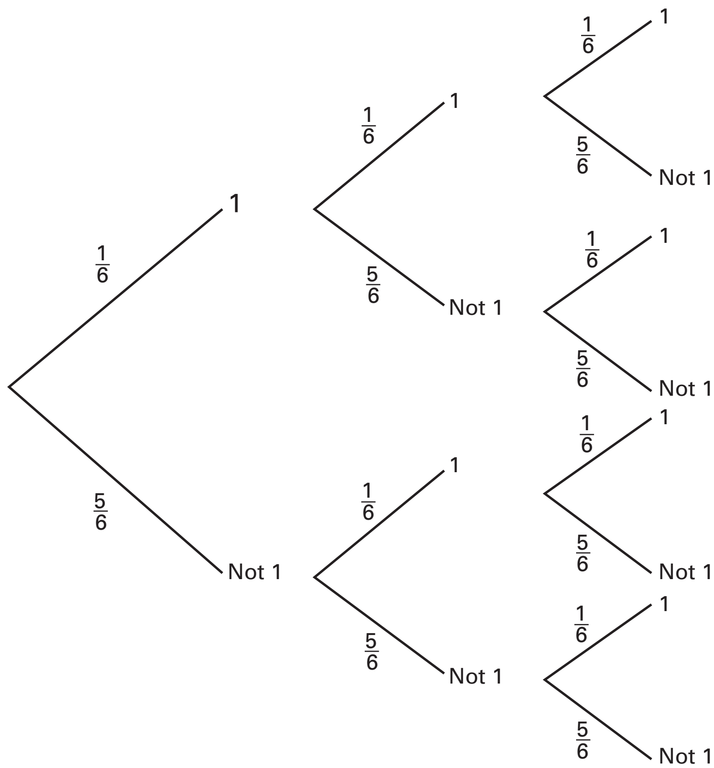
2. Explain what Celia did to make this tree.

Josh says, "There is only one route in this tree with 'not 1' for each number cube, so the chance of rolling 'not 1s' with three number cubes is one out of eight."

Celia says,"No, Josh, I don't think so. There are actually 5 × 5 × 5 or 125 routes that do not have a '1' in them."

3. a. Is Josh wrong? Explain your thinking.

   b. Explain how Celia reasoned to find that there are 5 × 5 × 5 routes for "not 1" with any of the number cubes.



If you want Celia's tree diagram to be easily understood, some extra information is needed. This makes the tree diagram into a **chance tree**.

4. a. **Reflect** What is the difference between a tree diagram and a chance tree?

   b. Use the chance tree to explain that the chance of rolling three 1s with three number cubes is $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$.

   c. Write the chance of rolling three 1s as a decimal.

   d. Calculate the chance of rolling "not 1s" with three number cubes.

   e. **Reflect** Do you think choosing to roll three number cubes in the game of Hog is a good decision? Explain your thinking.

5. a. Investigate the chance of rolling "not 1s" for another number of number cubes in Hog.

   b. If you want the biggest chance of not getting a "1," how many number cubes should you roll? Explain your reasoning.

   c. If you want to get a big score, how many number cubes do you think you should roll? Explain your reasoning.

A School Club Meeting

Sonia and Aysa are cousins. They both live in Middletown, but they go to different middle schools. Both Sonia and Aysa are members of the committee that organizes the clubs at their respective schools. All middle schools in Middletown will send representatives from their club committees to a citywide meeting.

Sonia's committee has five students. Two of them can go to the meeting.

Aysa's committee has three students and will send one of them to the meeting.

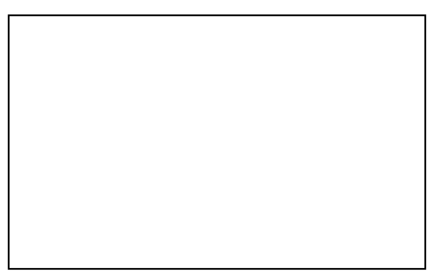
The two committees decide to select at random the students who will go to the meeting.

6. a. What is the chance that Sonia will be selected?

   b. What is the chance that Aysa will be selected?

Sonia wonders what the chance is that both she and Aysa can go to the meeting. To find out, she makes a diagram.

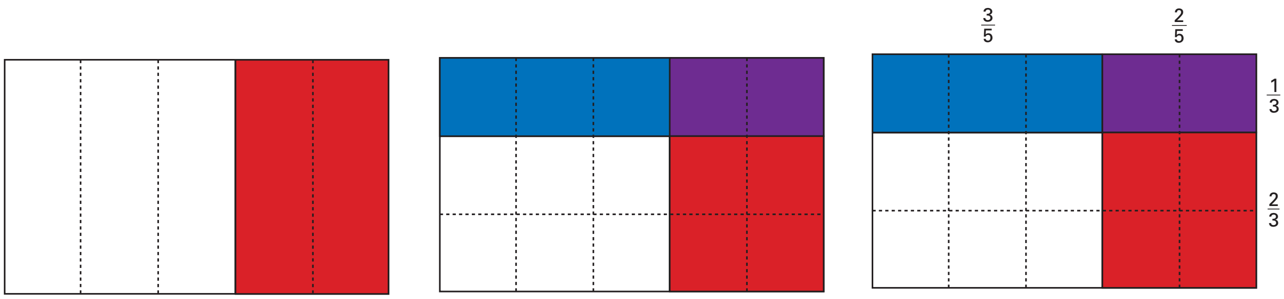
First she draws a rectangle.



She divides it in five equal vertical strips and shades two of the strips.

She divides the whole rectangle in three equal horizontal strips and shades one. Note that she has to shade all the way across.

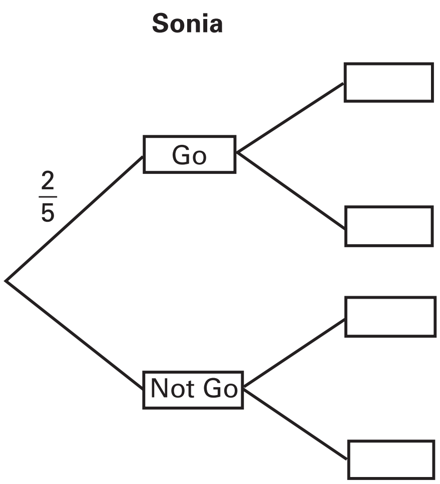
Finally she writes some fractions with the diagram.



7. a Explain how each of Sonia's diagrams relates to finding the chance that both of them will go to the meeting.

b. What is the chance that both Sonia and Aysa will go? How did you find this chance?

c. What is the chance that neither of them will go to the meeting?



The rectangle Sonia drew to help her figure out the chance is called an **area model**. You can also use a chance tree to calculate the chance that both Sonia and Aysa will be selected to go to the meeting.

Here you see the beginning of the chance tree you might use.

8. a. Copy the chance tree and finish it.

b. Calculate the chance that both Sonia and Aysha will go to the meeting. Write the chance as a fraction as well as a percentage.

c. What is the chance that only one of them will go to the meeting? Write the chance as a fraction as well as a percentage. Explain how you found this chance.

Sonia and Aysa have a friend named Dani. Dani is also on the club committee at his middle school.

Four students are on his committee, and one of them will be sent to the meeting to represent the committee.

9. a. Can you use the area model to find the chance that Sonia, Aysa, and Dani will all be sent to the meeting? If so, show how to do this. If not, explain why not.

b. Can you use the chance tree to find this chance? If so, show how to do this. If not, explain why not.

Tests

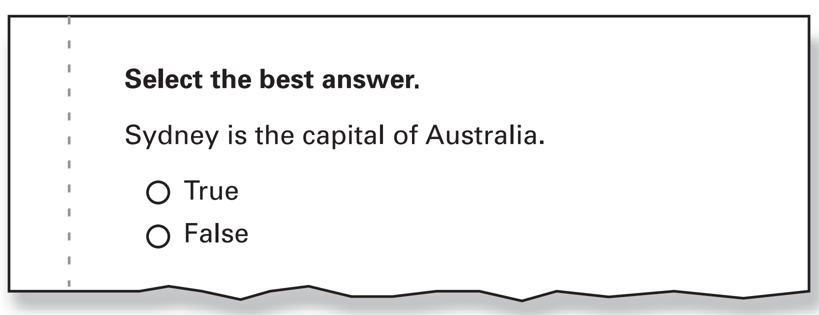
In Section C, you explored the chances of guessing correctly on a true-false test by flipping a coin. You estimated the chance based on a simulation. In this case, you can also calculate the theoretical chance.

10. a. Make a chance tree that you can use to calculate the chance of getting three true-false questions correct by guessing.

b. Use the tree you made for part **a** to find the following chances.

- The chance that two out of the three questions are answered correctly.
- The chance that all questions are answered incorrectly.
- The chance that at least one of the questions is answered correctly.

In Section C, you worked on problems about Charlie who took a geography test consisting of ten statements that are either true or false.



Charlie guessed the answers to all questions. He will pass the test if he got at least seven answers correct.

11. a. Describe what a chance tree for the geography test that Charlie took by guessing would look like. Note: You do not need to draw the whole chance tree, but you may want to draw a part of it.

b. What is the chance that Charlie will guess all 10 questions correctly? How did you calculate this chance?

As part of a test, Jamie needs to answer two multiple-choice questions. Each question has three possible answers, labeled A, B, and C.

Jamie says, "Last time, I guessed all the answers and did not pass the test. I think I can do better this time."

12. a. Use a chance tree to find the chance that Jamie will guess both questions correctly.

b. What is the chance that Jamie will guess only one of the two questions correctly?

c. Describe an easy way to find the chance that Jamie will guess both questions incorrectly.

d. **Reflect** Is it possible to use the area model to find these chances? If yes, show how. If no, explain why not.

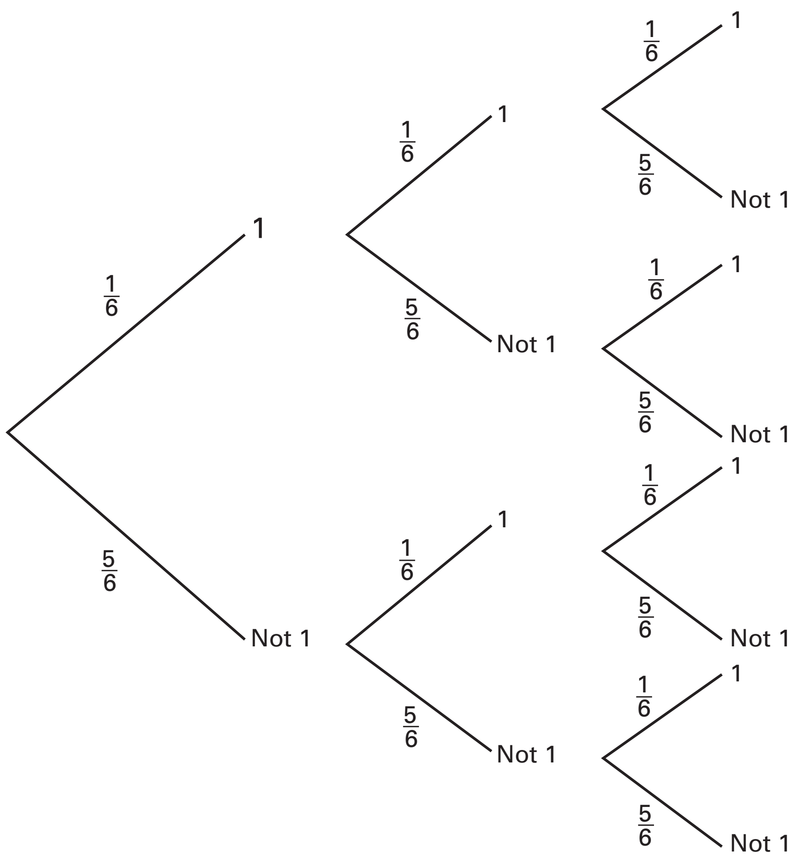
D Computing Chances

Summary

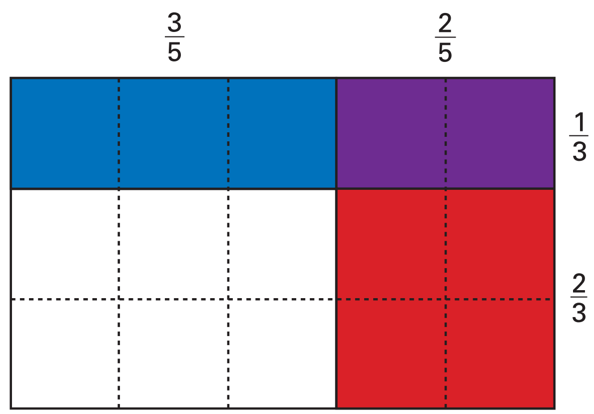
To find if the chance that an event will occur in different ways, you can collect data from a survey. Sometimes you can compute the theoretical chance of an event.

Chance: If all possible outcomes are equally likely, the chance that an event will occur is the number of successful outcomes divided by the number of possible outcomes.

With a chance tree, you can calculate chances for combined events. To find the theoretical chance, you have to carefully count the possibilities in which you are interested.



Sometimes an area model can be used to solve a chance problem about combined events.



The advantage of a chance tree over an area model is that you can combine more than just two outcomes.

Check Your Work

1. a. Use the chance tree for the game of Hog, in problem 3 of this section, to calculate the chance of rolling two 1s with three number cubes. Write the chance as a fraction and as a percent.

   b. What is the chance, as a percentage, of rolling "not 1s" with six number cubes in the game of Hog?

2. Mr. and Mrs. Lewis have four daughters. You may assume that the chance of having a son is the same as having a daughter: $\frac{1}{2}$. Comment on each of the following statements.

   a. The chance that their next child is a girl is smaller than $\frac{1}{2}$ because a family with five daughters is very unlikely.

   b. The chance is one half because the chance of a girl is $\frac{1}{2}$.

   c. The chance is larger than $\frac{1}{2}$ because the Lewises apparently have a tendency to have girls.

Remember Sonia and Dani from problems 6–9 in this section? Sonia's committee has 5 students, and two of them can go to the meeting. Dani's committee has 4 students, and one of them will be sent to the meeting to represent the committee.

3. Use the area model to calculate the chance that both Sonia and Dani will go to the meeting.

River Middle School has lockers with three-digit codes. The school wants to install lockers with two-letter codes.

4. **a.** Are there more combinations with three digits or with two letters?

b. If you were in charge at a school, would you choose lockers with three digits or two letters, assuming that they have the same quality and price? Explain your reasoning.

c. Mae is a student at River Middle School and does not have her two-letter code yet. What is the chance that she will get RR as her code?

For Further Reflection

Describe the different ways you have used to find the chance of an event in all of the sections in Second Chance. Can you use any way you want for any situation? Explain why or why not.

Additional Practice

Section A Make a Choice

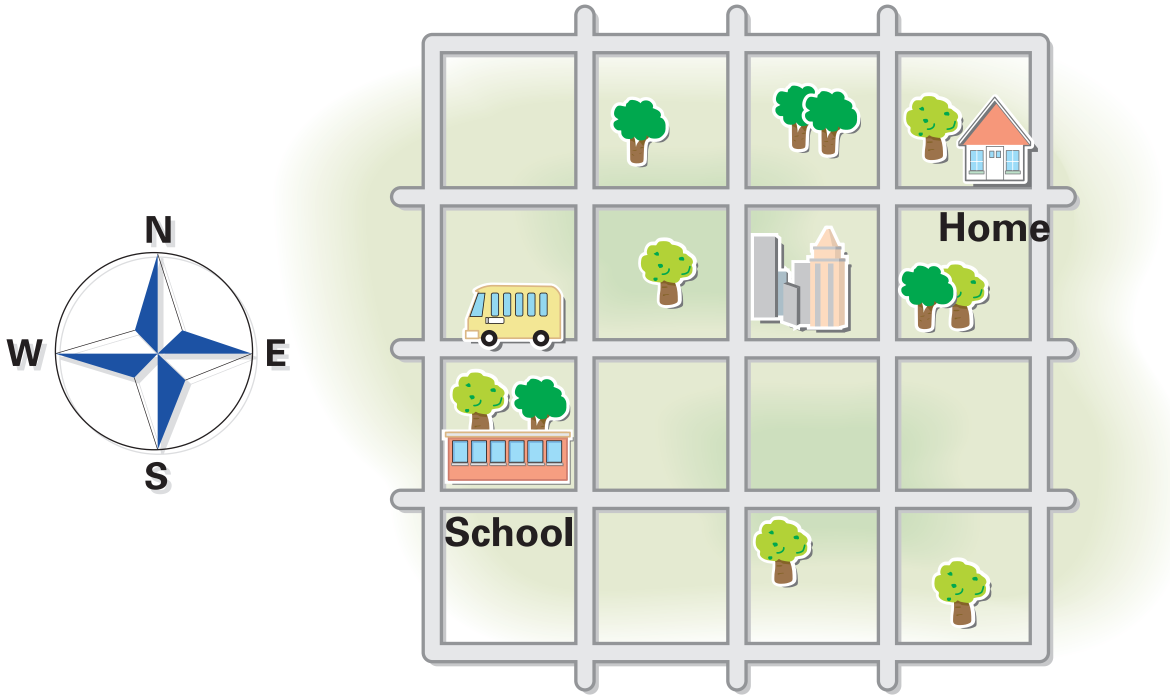
1. To decide whether to play the game Hilary wants to play or the game Robert wants to play, they toss a coin three times. They will play the game Hillary wants if there are exactly two heads.

   a. Do you think this is a fair way of deciding?

   b. How many different outcomes are possible? Explain your reasoning.

   c. Make a tree diagram of tossing three coins.

   d. What is the chance for the outcomes of tossing exactly two heads?



2. Robert walks home from school; both places are at the corner of two streets and the map looks like a grid.

   a. How many blocks north does Robert have to walk? And how many east?

   b. How many different routes can Robert take home from school?

Section B A Matter of Information

Source: 2003 National Assessment of Educational Progress

This graph is based on the results of a survey about how often students from grades 4 and 8 read for fun.

Suppose you randomly pick a student from this survey from the fourth grade and one from the eighth grade.

1. a What is the chance that the eighth grade student will read almost every day?

b. Compare the chances that each of them will read almost every day.

c. Is the chance of the eighth grader reading less than once or twice a week greater than the chance the fourth grader doing the same? Explain how you found out.

2. The Edwards Middle School newspaper wanted to report on the grades of students who were band members. The results of a survey of everyone in the school are in the table.

	Average B or Higher	Average Less Than B	Total
Band Member	27	22	
Non-Band Member	115	108	
Total			

a. Complete the table.

b. A student is chosen at random from the school.

- What is the chance that the student is in the band?
- What is the chance that the student has a B or higher grade?
- What is the chance that the student is in the band and has a B or higher grade?

c. What is the chance that the student is in the band and has a grade less than a B?

d. Assuming the student is in the band, what is the chance that he or she has a grade less than a B?

e. What is the difference between the questions in parts **c** and **d**?.

3. A survey of 140 seventh grade students at Bell Middle School was given. These are some of the results.

- 62 of the students played video games two or more hours a day.
- 45 of the students played video games two or more hours a day and liked school.
- The chance that a randomly chosen student from the survey likes school is $\frac{2}{3}$.

	Like School	**Do Not Like School**	**Total**
Play Video Games 2 or More Hours a Day			
Play Video Games Less Than 2 Hours a Day			
Total			**140**

a. Copy and fill in the table with the correct numbers of students, using the results from the survey above.

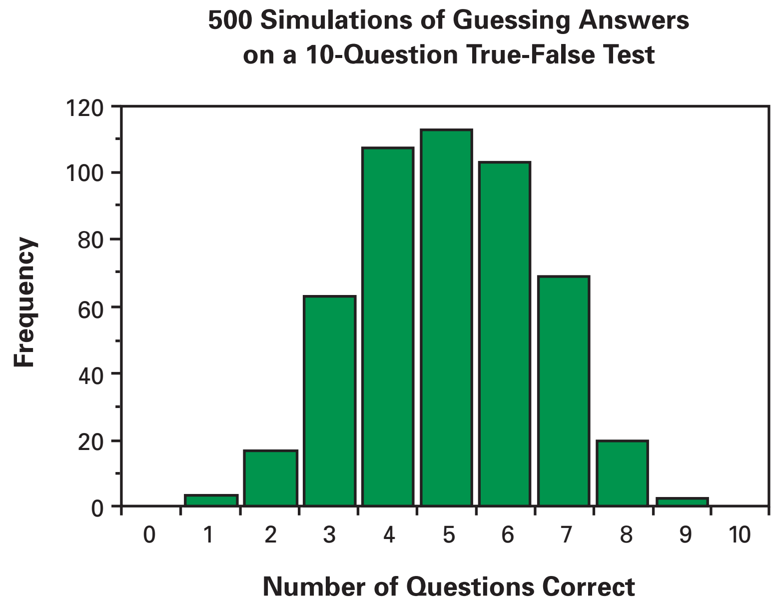
b. What is the chance that a randomly chosen seventh grader plays video games less than two hours a day and likes school?

c. If you know that a student likes school, what is the chance that he or she plays video games less than two hours a day?

d. Explain how parts **b** and **c** are different.

Section C In the Long Run

A graphing calculator was used to simulate guessing the answers on a ten-question true-false test. The calculator simulated taking the test 500 times.



The graph shows the results of the simulation. You pass the test when seven or more out of the ten questions are correct.

1. a. What is the most likely outcome? Why do you think so?

b. Estimate the chance that you would pass if "passing" were changed from 7 to 6 out of the 10 correct.

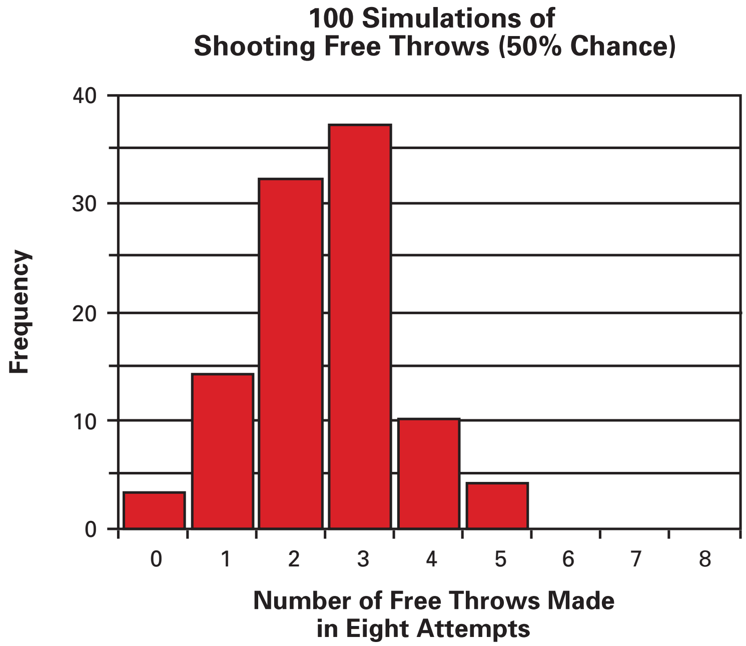
c. Estimate the chance that you would get nine or more correct by guessing.

2. Sabrina has a 50% free-throw shooting percentage. She wants to know the chance that she has of making at least three free throws in the next eight shots she takes.

a. Describe how she might use a simulation to help her find out.

b. Would her chances of making at least three free throws in the next six tries be better than her chance of making at least three in the next eight tries? Why or why not?

3. The graph shows the results of a simulation of 100 tries of eight free throws each, where the player has a 50% chance of scoring on each free throw.



   a. Estimate the chance that a player with a 50% chance of scoring a free throw will make three free throws in the next eight tries.

   b. Estimate the chance that the player will make at least three free throws in the next eight tries.

   c. Is the chance that the player will make at least three free throws in the next eight tries greater than or less than the chance of making less than three? Explain your reasoning.

4. Jesse wrote four statements about chance on the board. Which ones do you think are true and why?

   a. The chance of getting H T T T H when you toss a coin is smaller than the chance of getting T H T H T.

   b. The chance of getting all heads in five tosses of a coin is the same as the chance of getting all tails.

   c. The chance of getting at least six out of ten questions by guessing correctly on a ten-question test is the same as 1 minus the chance of getting up to five questions correct by guessing.

   d. If you get a long string of heads in a row when you toss a coin, the chance that you will get a tail next is more than 50%.

Section D Computing Chances

1. All the students in Jamie's school will get a new code of three digits for their lockers. How can you compute, in a smart way, the chance that her code will be 2–5–6?

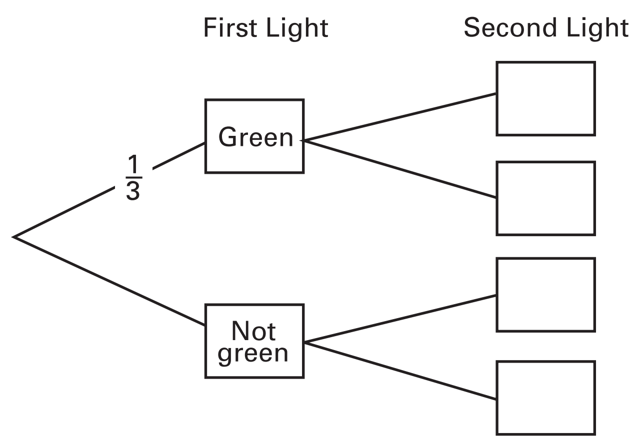
2. Matthew has a 75% free-throw shooting percentage. He wants to know the chance he has of making both free throws if he takes two shots.

   a. Calculate this chance. Explain how you found your answer.

   b. What is Matthew's chance of missing at least one shot if he takes four shots? Explain how you found your answer.

3. Hillary rides her bike to school. There are two traffic lights on the way. By keeping track of how often the lights are green when she gets to them, she has found out that the first traffic light is green around $\frac{1}{3}$ of the time and the second about $\frac{1}{4}$ of the time.

   She makes the following chance tree to compute the chance that she has to stop twice on her way to school.



   a. Finish the chance tree.

   b. What is the chance that she has to stop twice on her way to school?

   c. What is the chance that she has to stop only once?

   d. Draw an area model for the problem of the two traffic lights.

Answers to Check Your Work

Section A Make a Choice

1. a. Without drawing a tree diagram, you could write down all possible class trips, but this would take some time. You could also reason that you can choose one of four lakes, and for each lake you can either choose to camp or to stay in a lodge, so you now have 4×2 or 8 possibilities. Each of these 8 possible trips can have either a boat or a bus trip. So now you have 8×2 or 16 possible class trips from which to choose. In short, the number of possible trips is $4 \times 2 \times 2 = 16$. It might help to think about the branches in the tree.

   b. Robert is right. In the tree, 8 out of the 16 trips have a boat tour. So the chance is $\frac{8}{16}$. Noella is right as well, because half of the trips have the bus tour, and the other half have the boat tour. So the chance they will go on a boat trip is $\frac{1}{2}$, which is 50%. Of course you can also see that the $\frac{8}{16}$ is equal to $\frac{1}{2}$ and to 50%, so now you know that Noella is right, too.

2. No, Mario's advertisement is not correct. To find out how many three-course meals Mario's serves, you can, for example, draw a tree diagram—with 2 appetizers, 4 main courses, and 3 desserts from which to choose—and count all possible endpoints.

   Or you can list all of the possible meals.

   You can also reason the way you did for problem 1.

   All these methods will lead to the fact that Mario serves $2 \times 4 \times 3 = 24$ different three-course meals.

   There are many ways to make over 30 different three-course meals. For example, Mario's could have one more appetizer, which would make $3 \times 4 \times 3 = 36$ meals.

3. a. The chance that Diana has a meal with soup and beef is 3 out of 24, or $\frac{3}{24}$, or $\frac{1}{8}$. You can use a tree to find this by counting the meals with soup and beef. You can also reason about how many different meals have soup and beef. You only have a choice for dessert. So three different meals (one for each dessert) are possibilities with both soup and beef out of the 24 possible meals.

b. The chance Diana has a meal without fish is $\frac{3}{4}$ or 75%. You can find this answer in different ways. For example, you can find how many meals have fish, that is 1 out 4 because there are 4 main courses. So 3 out of 4 do not have fish.

You can also use a tree diagram and count all meals without fish, There are 18 out of 24, so the chance is $\frac{18}{24}$, which is $\frac{3}{4}$.

4. a. No, Diana is not correct. There are 8 meals that have pudding for dessert out of 24, so the chance she will pick a meal with pudding for dessert is $\frac{8}{24}$ or $\frac{1}{3}$. You can also reason about desserts only: there are 3 options for dessert, so $\frac{1}{3}$ of all possible three-course meals will have pudding. You can also use the tree diagram and count all meals with pudding.

b. 16 surprise meals are possible if pudding cannot be chosen for dessert. There are two options left for dessert: fruit and ice cream, so there are now $2 \times 4 \times 2 = 16$ possible meals.

Section B A Matter of Information

1. a. The chance that a randomly chosen doctor is female is $\frac{210{,}000}{840{,}000}$, which is about $\frac{2}{8}$ or 25%. You can find the percentage using your calculator or you can estimate.

b. It seems like there is a difference. Sample answer: The chance that a doctor would be female rather than male many years ago was smaller. For the group of doctors over 35 years of age, only 150,000 out of 700,000 are female, which is $\frac{15}{70}$, or about 20%. For the group of doctors under 35 years old, the chance that a doctor is female is 60,000 out of 140,000, or a little over 40%.

c. The chance that a randomly chosen doctor from a set of doctors you know to be under 35 will be a male is 80,000 out of 140,000, or $\frac{8}{14}$ (use the data in the first row of the table), which is a little over 60% (59% is O.K. too). You might also reason from the work in **b** that about 40% were female, so the chance of a male doctor would be 100%—40% or 60%.

d. The chance a doctor chosen at random from the ones that are male will be older than 35 is $\frac{550{,}000}{630{,}000}$, or $\frac{55}{63}$, which is about 80% (or 79%).

e. Answers will vary. Sample answers:

- Based on the data in the table, if you chose a doctor at random, the most likely outcome will be a male doctor 35 years old or over. This has a chance of about 66%.
- Based on the data in the table, the chance that a randomly chosen doctor is a female under 35 years old is only about 7%.
- It looks like younger doctors are more balanced—male/female, though a few more are male, but older doctors are mostly male.

2. Different answers are possible. Sample answer: She probably had to replace the E, T, and A, since these are the most commonly used letters in the English language. You can find information on frequently used letters in the table in this section or on the Internet (also for other languages).

3 a. The finished table will look like this:

	Saw Movie	Did Not See Movie	Total
Middle School	34	66 (100–34)	100
High School	86 (120–34)	14 (80–66 or 100–86)	100 (200–100)
Total	120	80 (200–100)	200

The rows and columns all have to add up properly. You can start filling in the middle numbers in the first and last row and the first and last column. Using two of these numbers, you can fill in the number in the middle.

b. Answers will vary. You can write different correct statements. For example:

- Of the middle school students, most (about $\frac{2}{3}$) did not see the movie.
- Of the high school students, most (86%) did see the movie.

c. Of all the students, 80 did not see the movie, so you only look in the second column in the table. The chance that a student who told you he hadn't seen the movie is in middle school is 66 out of 80, which is about 80% (82.5%).

4 a. The chance that the blood type of a randomly selected person is B is 10%. This is 8% for B positive and 2% for B negative. You can only add the percents because the categories do not overlap in any way. If there were 100 people, 8 of them would be B positive and 2 B negative, so 10 of the 100 or 10% would have type B blood.

b. You can reason the same way about the percents. The chance that the Rhesus factor of a randomly selected person, is positive is 36% + 38% + 8% + 3.5% = 85.5%.

c. It would change a little. If you know a randomly selected person has type B blood, you don't look at any of the other blood types. Then the chance that the Rhesus factor is positive is 8 out of 10, which is 80%

Section C In the Long Run

1 a. This game is fair since each player has the same chance of rolling each outcome. Of course, one player can have more luck than the other one, but if the game is played for a long time, both players will end up with the same result.

b. This game is not fair. A has a bigger chance of winning. You may need to play the game to find this out. You can also reason: there are only 6 ways to get doubles and 30 ways to get "not doubles." You can find the chance for rolling a double with two number cubes from the chart in Section A. So the chance of doubles is less than the chance of "not doubles," which means that A (who wins if no double occurs) has a bigger chance of winning.

2 a. If you roll a number cube a thousand times, you would expect a 6 to come up in about $\frac{1}{6}$ of the rolls, so about $\frac{1000}{6}$ = 167 times.

b. The numbers divisible by three on a number cube are 3 and 6, which is one third of the numbers on the number cube. If you roll a number cube 100 times, you expect a number divisible by three to come up in about $\frac{1}{3}$ of the rolls. This is about 33%, or about 33 times out of 100.

3 a. Answers can vary. Here are some correct and incorrect answers.

- "No, I do not agree with Jody because on each roll the chance of rolling a 6 stays the same; it is $\frac{1}{6}$. The number cube has no memory."(correct)

- "Yes, I agree with Jody because if you roll a lot of times, there need to be 6s." (Over many, many times, you will have 6s, but 10 is not a lot of times.)
- "No, I don't agree with Jody. She might not have gotten some of the other numbers either, and they would be just as likely to show up as a 6. (Not quite right because all of the numbers, including 6, are equally likely to show up.)

b. Different answers are possible. You cannot decide this on only 10 rolls. Try it yourself 10 times and see how many rolls it takes to get a 6.

4 a. If Peter guesses the answer to this question, his chance of guessing it wrong is 2 out of 3, or $\frac{2}{3}$. Only one of the options is the correct answer, so the other two are wrong.

b. You can model guessing the answer to all five questions on the test by using a number cube as follows: Let two of the numbers (for example, 1 and 2) of the number cube mean that you guessed a question correctly; the other four numbers (3, 4, 5, and 6) mean that you guessed incorrectly. Now you roll the cube five times, once for each question, and you record whether you "guessed" correctly or incorrectly.

c. The chance that Peter has 3 or more questions on the quiz correct by guessing is $\frac{12}{50}$, which is 24%.

Section D Computing Chances

1 a. The chance of rolling two 1s with three number cubes is $3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$, which is about 7%. You can find this chance by following the paths in the chance tree that have two 1s and one "not 1." The chance of each of these outcomes is $\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$, and there are three such paths.

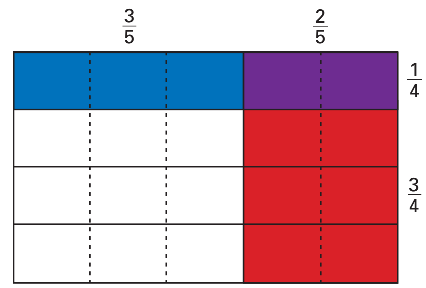
b. The chance of rolling no 1s with six number cubes in the game of Hog is $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$, or $(\frac{5}{6})^6$ = 33%. You can think about how a chance tree for six number cubes would look by extending the one from problem three. You then follow the path along the "not 1s" and multiply the chances.

2 a. This statement is not true. The chance that their next child is a girl is still $\frac{1}{2}$ because for each child the chance that it is a boy is the same as the chance that it is a girl. For a family to have five daughters seems unlikely, but it is $(\frac{1}{2})^5$, or $\frac{1}{32}$, which is about 3%, so about three out of every 100 families with five children are likely to have five girls.

b. This statement is true.

c. "The chance is larger than $\frac{1}{2}$ because the Lewises apparently have a tendency to have girls." If you believe that the chance of having a boy is the same as the chance of having a girl, this would not be true.

3. The chance that both Dani and Sonia will go is represented by the two purple squares. The chance is 2 out of 20, which is $\frac{1}{10}$, or 10%. You can also find this chance by calculating: $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$.



In this calculation $\frac{2}{5}$ is the chance that Sonia can go and $\frac{1}{4}$ is the chance that Dani can go.

4. a. If you write the two-letter code like this __–__ , you can make $26 \times 26 = 676$ combinations. Because the alphabet has 26 letters, you can choose one to fill in the first space, and you can fill in the second blank with any letter, so the total is 26×26.

With three numbers, the code can be written like this: __–__–__. For each "place," you can choose from 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), so there are $10 \times 10 \times 10 = 1{,}000$ possible codes. This is more then 676, so there are more three-number codes.

b. Different answers are possible and correct. Be sure to give a good reason for your choice. You may want to discuss your solutions with your classmates. What you choose may depend on the number of students and lockers. If 676 lockers is enough, you can choose a two-letter code. You may want to have more lockers later and prefer the three-digit code. You may also consider what is easier for the students to remember, three digits or two letters.

c. If the codes are assigned at random from all possible two-letter codes, the chance that Mae will get RR as her code is 1 out of 676; this is about 0.0015, or 0.15%.